Q1

interior: $\{z \mid 0 \leq |z| < 1\}$

exterior: $\{z \mid |z| > 1,\ z \neq 2\}$

boundary: $\{z \mid |z| = 1\} \cup \{2\}$

Q2

① $\emptyset$: open - vacumn true $\qquad$ $\mathbb{C}$: open - by definition

$\emptyset$ and $\mathbb{C}$ are closed.

② $\emptyset$ is closed and unbounded.

Q3

neither open or closed.

Q4

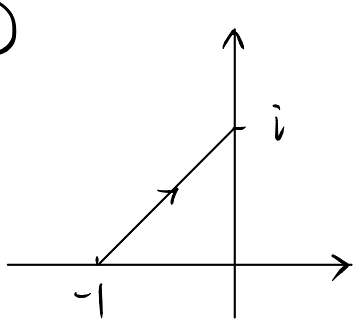
let $x = \dfrac{z+\bar{z}}{2}$, $y = \dfrac{z-\bar{z}}{2}$

$$f(x,y) = \frac{\bar{z}-1}{z\bar{z} - z - \bar{z} + 1} = \frac{\bar{z}-1}{(z-1)(\bar{z}-1)} = \frac{1}{z-1}$$

so $\dfrac{\partial f}{\partial \bar{z}} = 0$. since no $\bar{z}$ exist. f is holomorphic except for $z=1$.

Q5

①



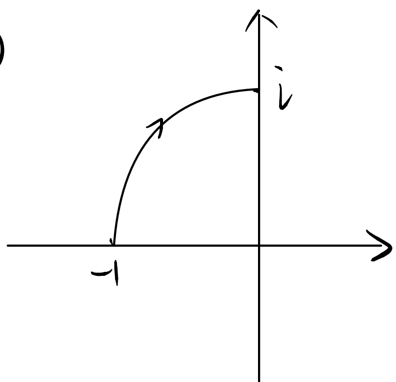
line : $y = x + 1$

$$\gamma(t) = -1 + (1+i)t$$

$$\gamma'(t) = 1 + i$$

$$\int_C f(z)\,dz = \int_0^1 (2t^2 - 2t + 1)(1+i)\,dt = \frac{2}{3}(1+i)$$

②



arc : $e^{i\theta}$ θ from π to $\frac{\pi}{2}$

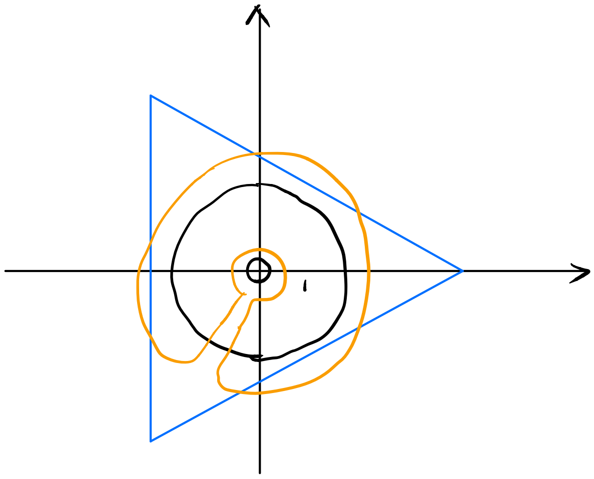
$$\gamma(\theta) = e^{i\theta}$$

$$\gamma'(\theta) = ie^{i\theta}$$

$$\int_C f(z)\,dz = \int_{\pi}^{\frac{\pi}{2}} ie^{i\theta}\,d\theta$$

$$= 1 + i$$

Q6



no simple-connected shape, such that containing the ◯ but not ∘.
∵ ∘ is an interior point of ◯

Q7

Let $f(x) = e^{z^2}$ ∠ By Cauchy's integral formula,

$$f(2) = \frac{1}{2\pi i} \oint_C \frac{e^{z^2}}{z-2} dz = e^4$$

$\Rightarrow 2\pi i e^4$

Q8

By cauchy's integral theorem. 0.

Q9

$-2\pi i f(2) - 2\pi i f(2)$

$= -4\pi i f(2)$