

Independence

(This question is more for fun.)

1. **Bivariate random variable:**

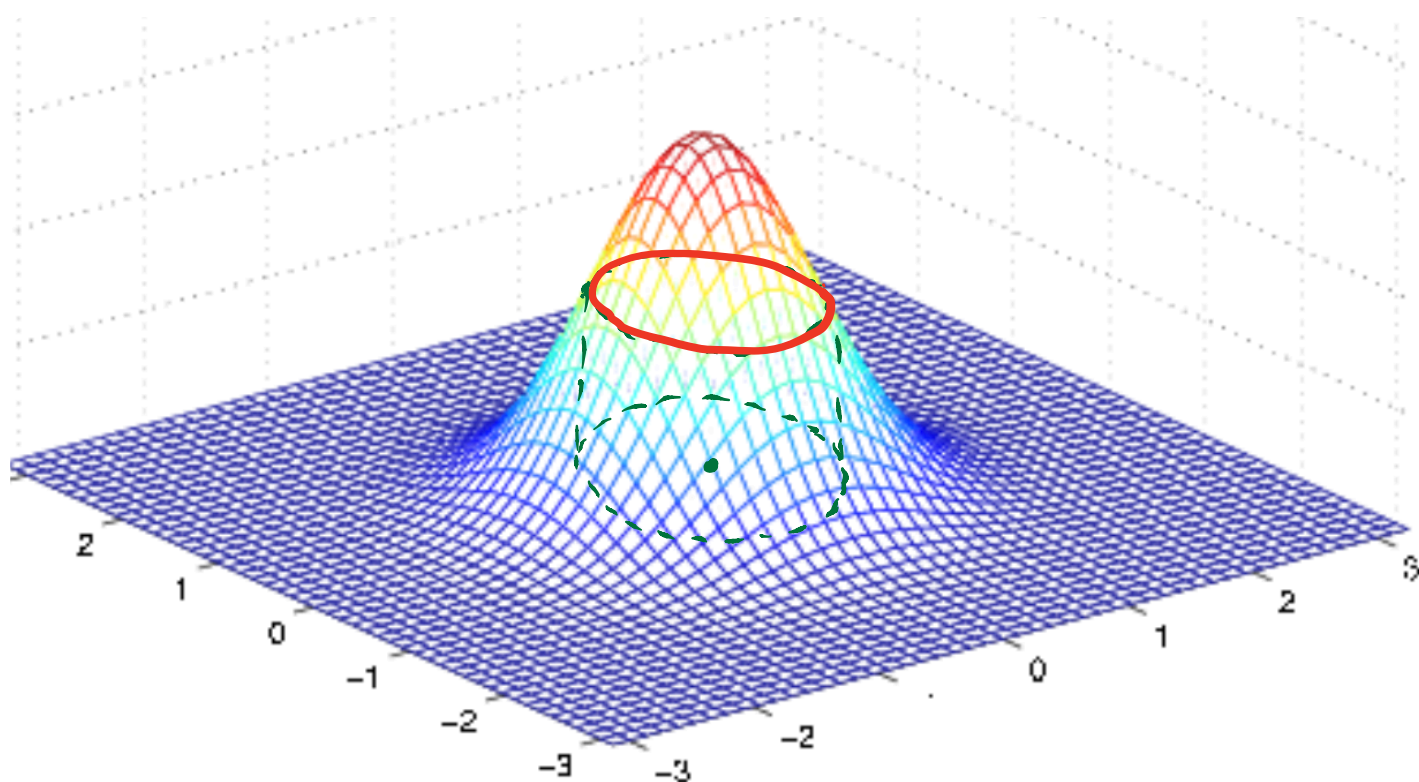
$$f_{XY}(x, y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}} e^{-\frac{1}{2(1-\rho^2)} \left[\left(\frac{x-\mu_X}{\sigma_X}\right)^2 - 2\rho\left(\frac{x-\mu_X}{\sigma_X}\right)\left(\frac{y-\mu_Y}{\sigma_Y}\right) + \left(\frac{y-\mu_Y}{\sigma_Y}\right)^2 \right]}$$

X and Y are independent $\Leftrightarrow \rho = 0$

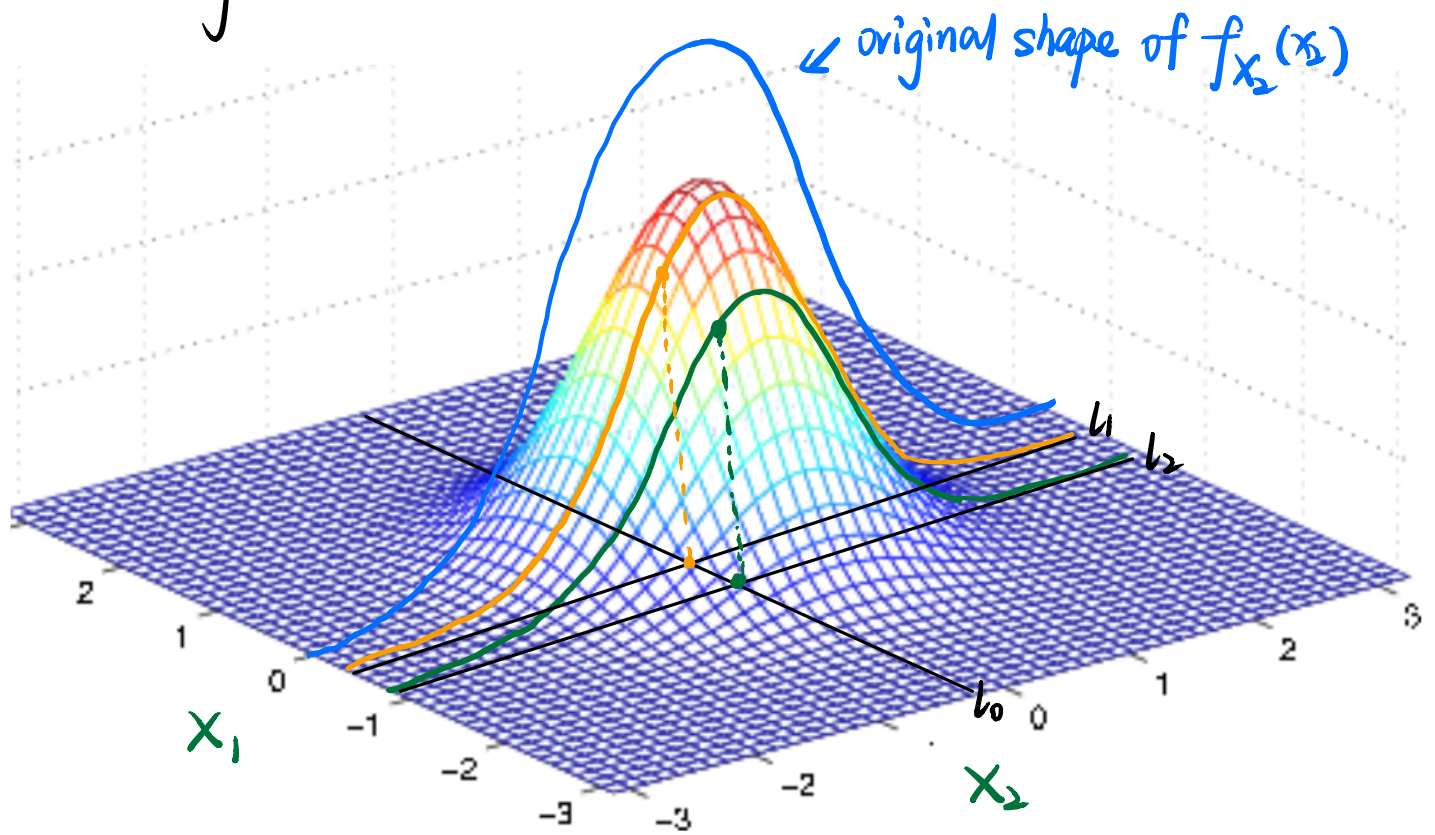
and the above graph is the standard case.

So: $f_{XY}(x, y) = \frac{1}{2\pi} \cdot e^{-\frac{1}{2}(x^2+y^2)}$

Then if x^2+y^2 is same, $f_{XY}(x, y)$ is same:



2. Generally:



Independence:

$$f_X(x_1, x_2) = f_{X_1}(x_1) \cdot f_{X_2}(x_2)$$

It's like you flatten the shape of $f_{X_2}(x_2)$ on height by $f_{X_1}(x_1)$ along the X_1 axis, and vice versa.

So if any two curves cut out by two lines parallel to X_1 or X_2 axis are not squeezed shapes of one another, X_1 and X_2 will be dependent. You can take the example used in Note 4 2.3 "visualization" as a simple example.

(More detailed explanation: if X_1, X_2 are independent, the with any fixed x_2 , and any randomly chosen x_{11}, x_{12} ,

$$\frac{f_X(x_{11}, x_2)}{f_X(x_{12}, x_2)} = \frac{f_{X_1}(x_{11})}{f_{X_1}(x_{12})} \quad \text{is just} \quad \frac{h_1}{h_2} = \frac{s_1}{s_2}$$

System.

1. Yes, your understanding is right.

The point is you define X_1, X_2, Y as the failure time, instead of reliability function directly.

2. Yes, you can use this method to solve other distributions.

But one thing you will find is : $Y = \min\{X_1, X_2\}$ has the same type distribution as X_1, X_2 is a special case for exp. distribution. Not all distributions satisfy this condition.